Lehrstuhl für Analysis und Modellierung

Lehrstuhl-Seminar
Wintersemester
2019/2020

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How to construct Breather Solutions Using Nonlinear Helmholtz Systems

16. Januar 2020 - 10:00
Seminarraum IADM 8.526, Pfaffenwaldring 57

Abstract:

This talk presents a new strategy to obtain time-periodic, spatially localized ("breather") solutions of cubic wave-type equations set on the full space. Its core point is the reduction to an infinite system of stationary Helmholtz-type wave equations.

In this way, we find breathers for the Klein-Gordon equation

$$\partial_t^2 U - \Delta U + m^2 U = \Gamma(x)\, U^3 \qquad \text{on } \mathbb{R}\times\mathbb{R}^3 \tag{KG}$$

using the ansatz $U(t,x) = \sum_k u_k(x) \mathrm{e}^{\mathrm{i}kt}$ where u_k are radial solutions of a Helmholtz system with $\sum_k \big\|(1+|\cdot|^2)^{1/2}\, u_k\big\|_\infty < \infty$. The key element is a detailed analysis of the oscillatory behavior of the components u_k in the far field, which will give rise to infinitely many distinct families of time-dependent solutions of (KG) bifurcating from any (given) time-independent solution of (KG).

Moreover, we sketch an idea to obtain "large" breathers $U \in L^p(\mathbb{R}\times\mathbb{R}^3)$, $2 < p \leq 4$, for the wave equation

$$\partial_t^2 U - \Delta U = \Gamma(x)\, |U|^{p-2} U \qquad \text{on } \mathbb{R}\times\mathbb{R}^3 \tag{W}$$

based on (dual) variational methods.

The first part of the talk is a result of the author's dissertation thesis (KIT, 2019); the second part provides a glimpse on current research.