



---

## Lehrstuhl für Analysis und Modellierung

Lehrstuhl-Seminar  
Sommersemester  
2021

**Dr. Ruoci Sun**

KIT Karlsruhe

### Complete integrability of the Benjamin–Ono equation on the multi-soliton manifolds

**16. September 2021 - 10:30**

**WebEx Meeting**

Abstract: This presentation, which is based on the work Sun [2], is dedicated to describing the complete integrability of the Benjamin–Ono (BO) equation on the line when restricted to every  $N$ -soliton manifold, denoted by  $UN$ . We construct (generalized) action–angle coordinates which establish a real analytic symplectomorphism from  $UN$  onto some open convex subset of  $\mathbb{R}^{2N}$  and allow to solve the equation by quadrature for any such initial datum. As a consequence,  $UN$  is the universal covering of the manifold of  $N$ -gap potentials for the BO equation on the torus as described by Gérard–Kappeler [1]. The global well-posedness of the BO equation on  $UN$  is given by a polynomial characterization and a spectral characterization of the manifold  $UN$ . Besides the spectral analysis of the Lax operator of the BO equation and the shift semigroup acting on some Hardy spaces, the construction of such coordinates also relies on the use of a generating functional, which encodes the entire BO hierarchy. The inverse spectral formula of an  $N$ -soliton provides a spectral connection between the Lax operator and the infinitesimal generator of the very shift semigroup. The construction of action–angle coordinates for each  $UN$  constitutes a firststep towards the soliton resolution conjecture of the BO equation on the line.

Keywords Benjamin–Ono equation, multi-solitons, global well-posed-ness, generalized action–angle coordinates, Lax pair, shift semigroup on Hardy space, universal covering manifold.

#### References

- [1] Gérard, P., Kappeler, T. On the integrability of the Benjamin–Ono equation on the torus, *Commun. Pure Appl. Math.* 74 (2021), no.8, 1685–1747, <https://doi.org/10.1002/cpa.21896>, 2021.
- [2] Sun, R. Complete integrability of the Benjamin–Ono equation on the multi-soliton manifolds, *Commun. Math. Phys.* 383, 1051–1092 (2021). <https://doi.org/10.1007/s00220-021-03996-1>