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Well-posedness and long-wave KdV approximation of hydrodynamic Gross-Pitaevskii equations

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Abstract:

We consider the Gross-Pitaevskii equation

$$i\partial_t q + \partial_{xx} q - 2(|q|^2 - 1)q = 0, \quad (1)$$

where $q(t, x) : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}$ represents an unknown wave function, subject to the boundary condition at infinity $\lim_{|x| \rightarrow \infty} |q(t, x)| = 1$. It can be put into a hydrodynamic form via the Madelung transform

$$\mathcal{M}(q) = (\rho, v) = \left(|q|^2, \operatorname{Im} \left[\frac{\partial_x q}{q} \right] \right). \quad (2)$$

We establish global-in-time well-posedness of the one-dimensional hydrodynamic Gross-Pitaevskii equations in the absence of vacuum in $(1 + H^s) \times H^{s-1}$ with $s \geq 1$ [1].

We furthermore discuss an approximative connection between the Gross-Pitaevskii and KdV equations in the long-wave regime, and formally extend it to the respective hierarchies.

[1] *Global-in-time well-posedness of the one-dimensional hydrodynamic Gross-Pitaevskii equations without vacuum.*

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