WEAK COUPLING ASYMPTOTICS FOR THE PAULI OPERATOR IN TWO DIMENSIONS

BAUR, MATTHIAS - UNIVERSITÄT STUTTGART

ABSTRACT. We compute asymptotic expansions for the negative eigenvalues of the Pauli operator in two dimensions perturbed by a weakly coupled potential with definite sign. Whereas previous results were limited to the case of radial magnetic fields and potentials, we are able to drop the assumption of radial symmetry entirely. For the proofs, we employ recent resolvent expansions by Kovařík and make extensive use of Schur complements to reduce the Birman-Schwinger operator corresponding to the perturbed Pauli operator.

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A THREE LINES ESTIMATE WITH AN APPLICATION TO THE LIEB-THIRRING AND CWIKEL-LIEB-ROZENBLUM INEQUALITIES

CARVALHO-CORSO, THIAGO - UNIVERSITÄT STUTTGART

ABSTRACT. In this talk I will present some recent improvements [1,2] on the best-known upper bounds for the optimal constants in the Lieb-Thirring (LT) and Cwikel-Lieb-Rozenblum (CLR) inequalities. These improvements were obtained by explicitly solving an interesting family of variational problems, whose connection with the LT and CLR inequalities has recently been established in [3] and [4], respectively. To obtain the explicit solutions, we first reformulate these variational problems as an interpolation-like inequality along three parallel lines in the complex plane, for certain spaces of holomorphic functions. We then compute the optimizers associated with these interpolation inequalities by combining some techniques of complex/Fourier and convex analysis.

References:

[1] Corso and Ried - On a variational problem related to the Cwikel-Lieb-Rozenblum and Lieb-Thirring inequalities
- https://arxiv.org/abs/2403.04347

[2] Corso - A generalized three lines lemma in Hardy-like spaces
- https://arxiv.org/abs/2407.10117

[3] Frank, Hundertmark, Jex, and Nam - The Lieb-Thirring inequality revisited
- https://ems.press/journals/jems/articles/666342

[4] Hundertmark, Kunstmann, Ried, and Vugalter - Cwikel's bound reloaded
- https://link.springer.com/article/10.1007/s00222-022-01144-7

WEYL FORMULAE AND CONCENTRATION OF EIGENFUNCTIONS FOR SINGULAR RIEMANNIAN METRICS WITH APPLICATION TO ACOUSTIC MODES IN GAS GIANTS

DIETZE, CHARLOTTE - LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ABSTRACT. We prove eigenvalue asymptotics of the Laplace-Beltrami operator for certain singular Riemannian metrics on Riemannian manifolds with boundary. We also describe the rate and profile at which eigenfunctions concentrate at the boundary. The model we consider is motivated by the study of propagation of soundwaves in gas planets. This is based joint work with Yves Colin de Verdière, Maarten de Hoop and Emmanuel Trélat and joint work with Larry Read.

CWIKEL-LIEB-ROZENBLUM TYPE INEQUALITIES FOR HARDY-SCHRÖDINGER OPERATOR

DUONG, GIAO KY - LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ABSTRACT. We discuss a Cwikel-Lieb-Rozenblum type inequality for the number of negative eigenvalues of the Hardy-Schrödinger operator, which takes the form of the Laplacian minus a potential term involving a Hardy weight and a function W, on $L^2(R^d)$. The bound is provided in terms of a weighted norm of W that is sharp in both large and small coupling regimes. We also derive a similar bound for the fractional Laplacian.

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CWIKEL-LIEB-ROZENBLUM TYPE ESTIMATES FOR THE PAULI OPERATOR

KOVAŘÍK, HYNEK - UNIVERSITÀ DEGLI STUDI DI BRESCIA

ABSTRACT. In this talk I will present an upper bound on the number of negative eigenvalues of a two-dimensional Pauli operator in terms of a weighted L^1-norm of the electric potential. It will be shown that the bound is sharp in both large and small coupling regimes. Based on a joint work with M. Baur.

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KATO'S INEQUALITY AND APPLICATIONS

KUSSMAUL, VALENTIN - UNIVERSITÄT STUTTGART

ABSTRACT. Kato's distributional inequality relates a magnetic Schroedinger operator, or its semi-group, to the corresponding object without magnetic field. We generalize these results to a setting of Hilbert-space valued L^2-functions and operator valued - that is, quantized - magnetic fields. This allows us to give a short and simple proof of pointwise exponential decay of confined states in non-relativistic quantum electrodynamics. The only previous proof of this result is based on the full machinery of stochastic integration in quantum field theory. This is joint work with Marcel Griesemer.

SPECTRAL PROPERTIES OF SOME FUNCTIONAL-DIFFERENCE OPERATORS

LAPTEV, ARI - IMPERIAL COLLEGE LONDON

ABSTRACT. We shall discuss Weyl type asymptotics of functional-difference operators associated to mirror curves. We also obtain some uniform inequalities for the Riesz means of the eigenvalues of functional-difference operators with decaying potentials. A number of open question will be presented.

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TITLE: TBA

PFEIFFER, PAUL - FERNUNIVERSITÄT IN HAGEN

ABSTRACT. TBA

ON THE CONCENTRATION OF LOW-LYING STATES IN THE 2D CONFINED STARK EFFECT

READ, LARRY - LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ABSTRACT. In this talk, I will consider the Schrödinger operator with a linear potential (Stark Hamiltonian) on a bounded 2D convex domain under Dirichlet boundary conditions. In the strong coupling limit, the eigenstates localise at the boundary point where the potential is minimised. My goal is to determine the asymptotic accumulation of states with energies below various orders of the coupling strength.

SPECTRAL ESTIMATES FOR THE RESOLVENT DIFFERENCE FOR SCHRÖDINGER AND LAPLACE OPERATORS UNDER SINGULAR PERTURBATIONS

GRIGORI ROZENBLUM - CHALMERS UNIVERSITY OF TECHNOLOGY

ABSTRACT. Let H_0 be the Laplace operator in $\mathbb{R}^d$ and V be a potential. If V is a bounded compactly supported function, the Schrödinger operator H_v is naturally defined, and *estimates for the eigenvalues* of the difference of resolvents, $(H_v - z)^{-1} - (H_V - z)^{-1}$, are easily found using the Hilbert formula

$$(H_v - z)^{-1} - (H_0 - z)^{-1} = -(H_v - z)^{-1}V(H_0 - z)^{-1}, \qquad (*)$$

and known since 1950-s. However, if V is a measure containing a singular component, for example, supported on a surface, formula $(*)$ breaks down. In the talk we describe a new approach to this and similar problems, with a singular measure V, producing order sharp eigenvalue estimates and, in some cases, asymptotics. Estimates are also found for eigenvalues of the difference of powers of resolvents and for resolvents of some boundary value problems for the Laplacian.

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Two-sided Lieb-Thirring bounds

Severin Schraven
Technical University of Munich

Abstract

We discuss upper and lower bounds for the number of eigenvalues of semi-bounded Schrödinger operators in all spatial dimensions. For atomic Hamiltonians with Kato potentials one can strengthen the result to obtain two-sided estimates for the sum of the negative eigenvalues. Instead of being in terms of the potential itself, as in the usual Lieb-Thirring result, the bounds are in terms of the landscape function, also known as the torsion function, which is a solution of $(-\Delta + V + M)u_M = 1$ in $\mathbb{R}^d$; here $M \in \mathbb{R}$ is chosen so that the operator is positive.

This talk is based on the preprint arXiv:2403.19023 which is joint work with S. Bachmann and R. Froese.

LIEB-THIRRING INEQUALITIES FOR THE SHIFTED COULOMB HAMILTONIAN

ZENG, ZHUOYAO - UNIVERSITÄT STUTTGART

ABSTRACT. In this talk we consider sharp Lieb-Thirring (LT) inequalities for
the family of shifted Coulomb Hamiltonians. We prove the classical LT in-
equalities with the semi-classical constant for this family of operators in any
dimension $d \geqslant 3$ and any $\gamma \geqslant 1$. Moreover, we prove that the semi-classical
constant is never optimal for the Cwikel-Lieb-Rozenblum (CLR) inequalities
for this family of operators in any dimension. In this case, we characterize the
optimal constant as the minimum of a finite set and provide an asymptotic
expansion as the dimension grows. Using the same method to prove the CLR
inequalities for Coulomb, we obtain more information about the conjectured
optimal constant in the CLR inequality for arbitrary potentials.

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