Course Description Stability of Nonlinear Waves – Summer 2021

Content. Traveling waves are solutions to nonlinear partial differential equations that propagate over time with a fixed speed without changing their profiles. These special solutions arise in many applied problems where they model, for instance, water waves, nerve impulses in axons or pulses in optical fibers. Therefore, the naturally associated question of their dynamic stability is of interest: do solutions whose initial conditions are small perturbations of the wave stay close to the original solution of not? Indeed, only those waves that are stable against (localized) perturbations are observed in practice.

The first step in the stability analysis is to linearize the underlying partial differential equation about the wave and compute the associated spectrum, which is in general a nontrivial task. To approximate the spectra of various waves, such as fronts, pulses and periodic wave trains, we introduce the following tools:
- Sturm-Liouville theory
- exponential dichotomies
- Fredholm theory
- the Evans function
- parity arguments
- exponential weights

The next step is to derive useful estimates on the linear solution operator, or semigroup, using the spectral information. A complicating factor is that any non-constant traveling wave has spectrum up to the imaginary axis. Finally, we employ the estimates on the linear solution operator to close a nonlinear iteration argument via the variation of constants formula, or Duhamel formulation.

Expected previous knowledge. Analysis 1-3. Some background in operator/spectral theory (functional analysis) and dynamical systems is beneficial, but not necessary.

Objective. After successful completion of this module students can explain the significance of traveling waves and their dynamic stability. They understand what the concept of stability entails, can outline the main steps in a stability analysis and can address potential complications. They comprehend the structure of the spectrum associated to a nonlinear wave and have acquired several mathematical tools to compute or approximate the spectrum. The students have mastered several techniques to derive (in)stability of the wave from spectral information. They understand how spectrum and stability might depend on the underlying space of perturbations.

Recommended reading. We will follow the (e-)book:

- Kapitula, Todd; Promislow, Keith. Spectral and dynamical stability of nonlinear waves. Applied Mathematical Sciences, 185. Springer, New York, 2013.

Students can access the e-book from within the network of the University of Stuttgart using SpringerLink: `https://link.springer.com/book/10.1007/978-1-4614-6995-7`

Exam information. The final grade is $\min\{0.7O + 0.3M, O\}$, where O is the grade for a 30 minute oral exam at the end of the semester and M is the grade obtained by working out and presenting a model problem during one of the (online) exercise classes.

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