GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
INTERNATIONAL ASSOCIATION of APPLIED MATHEMATICS and MECHANICS

6th GAMM Workshop on Analysis of Partial Differential Equations

September 19 – 21, 2018 - Universität Stuttgart

Scientific Program and Abstracts

Local Organizers:
Björn de Rijk
Wolf-Patrick Düll
Guido Schneider

SimTech Cluster of Excellence

Spectral Theory and Dynamics of Quantum Systems
GRADUIERTENKOLLEG 1838

mathematik physik fakultät8

Universität Stuttgart

Wednesday, 19.09.2018

13:30 – 13:55	Welcome
13:55 – 14:00	Opening
14:00 – 14:45	Angkana Rüland: Uniqueness, Stability and Single Measurement Recovery for the Fractional Calderón Problem
14:45 – 15:15	Chris Kowall: Shadow Limit Approximation for Reaction-Diffusion Systems
15:15 – 15:45	Björn de Rjik: Diffusive Stability of Planar Wave Trains in Reaction-diffusion Systems Against Nonlocalized Perturbations
15:45 – 16:15	Coffee Break
16:15 – 17:00	Li Chen: The Maximal Negative Ion of the Time-Dependent Thomas-Fermi and the Vlasov Atom
17:00 – 17:30	Timo Weidl: Edge Resonances in Elastic Media with Zero Poisson Coefficient
17:30 – 18:00	Joel Kübler: Nonradial Bifurcation for the Henon Equation

Thursday, 20.09.2018

9:00 – 9:45	Christof Melcher: Topological Solitons in Chiral Magnetism
9:45 – 10:15	Roland Schnaubelt: Decay of Quasilinear Maxwell Equations with Conductivity
10:15 – 10:45	Coffee Break
10:45 – 11:15	Tomas Dohnal: First Order Coupled Mode Equations for Wavepackets in Higher Dimensions
11:15 – 12:00	Sebastian Schwarzacher: Two Scale Solutions to Rate-Independent Systems
12:00 – 13:15	Lunch
13:15 – 14:15	Ulisse Stefanelli – Overview Talk: A Variational Resolution for Incompressible Navier-Stokes
14:15 – 14:45	Sebastian Throm: Long-Time Behaviour for Smoluchowski's Coagulation Equation with Cluster In- and Output
14:45- 15:15	Wolfgang Reichel: Real-valued time-periodic localized standing waves for a class of nonlinear wave equations
15:15 – 16:00	Coffee Break – Poster Session
16:00 – 17:30	**Sixths** meeting of the GAMM activity group “Analysis of PDEs
18:45 19:30	Guided tour through “Brauhaus Calwer-Eck” with beer tasting Dinner at “Brauhaus Calwer-Eck”

Friday, 21.09.2018

9:00 – 9:45	Björn Stinner: Derivation and Analysis of a Cell Blebbing Model
9:45 – 10:15	Maximilian Moser: Convergence of the Allen-Cahn Equation to Mean Curvature Flow with 90°-Contact Angle in 2D
10:15 – 10:45	Coffee Break
10:45 – 11:15	Patrik Knopf: Weak Solutions of a Cahn-Hilliard System with Dynamic Boundary Conditions
11:15 – 12:00	Helmut Abels: Diffuse Interface Models for Two-Phase Flows of Incompressible Fluids and Their Sharp Interface Limits
12:15	Closing

Venue

University of Stuttgart
Department of Mathematics
Room 8.122
Pfaffenwaldring 57
70569 Stuttgart

Conference Dinner

and guided tour through brewery (in German) with beer tasting
Brauhaus Calwer-Eck
Calwer Str. 31
70173 Stuttgart

Homepage:
http://calwereck.de/index.php

Poster:

Dominik Engl (Universiteit Utrecht)	Theories for incompressible rods and strings: a rigorous derivation via Γ-convergence
Elfriede Friedmann (Universität Heidelberg)	Coupled PDE-systems describing the physiological processes in the human eye
Tobias Haas (Universität Stuttgart)	Modulation equations at the Eckhaus boundary - The KdV equation
Max Hess (Universität Stuttgart)	Validity of the Nonlinear Schrödinger approximation for quasilinear dispersive systems
Bastian Hilder (Universität Stuttgart)	Modulating fronts for the Swift-Hohenberg equation with additional conservation law
Daniela Maier (Universität Stuttgart)	Breather solutions on periodic graphs

Invited Speakers:

Helmut Abels (Universität Regensburg)

Diffuse Interface Models for Two-Phase Flows of Incompressible Fluids and Their Sharp Interface Limits

Abstract: We will discuss so-called "diffuse interface models" for the flow of two viscous incompressible Newtonian fluids in a bounded domain. Such models were introduced to describe the flow when singularities in the interface, which separates the fluids, (droplet formation/coalescence) occur. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed in the model. Such kind of models became popular during the last decade for theoretical as well as numerical studies. We will give an overview of known analytic results on well-posedness and will discuss recent results on convergence of diffuse to sharp interface models.

Li Chen (Universität Mannheim)

The Maximal Negative Ion of the Time-Dependent Thomas-Fermi and the Vlasov Atom

Abstract: In this talk, I will briefly describe the results in ionization problem in stability of matter, present a generalized version Lieb's inequality which playes important role in getting the excess charge estimate. Then in the time dependent case, for both Thomas-Fermi and Vlasov Atom, we show an atom of atomic number Z described by the time-dependent Thomas-Fermi equation or the Vlasov equation cannot bind more than $4Z$ electrons. - This is a joint work with Heinz Siedentop.

Christof Melcher (RWTH Aachen University)

Topological Solitons in Chiral Magnetism

Abstract: Topological solitons in nonlinear field theories have attracted the interest of physicists, e.g. in the context of elementary particles, and mathematicians, e.g. in the context of large solutions to geometric PDE. A common goal is to understand the mechanisms which lead to the stabilization of such localized and topologically nontrivial field configurations. In this talk we shall focus on the effect of chiral symmetry breaking due to Lifshitz invariants featured in various condensed matter systems including in particular magnets without inversion symmetry, where the recent experimental discovery of so-called "chiral skyrmions" has raised the hope to use topology as a concept for future information technologies.

Angkana Rüland (Max Planck Institute for Mathematics in the Sciences Leipzig)

Uniqueness, Stability and Single Measurement Recovery for the Fractional Calderón Problem

Abstract: In this talk I discuss a nonlocal inverse problem, the fractional Calderón problem. This is an inverse problem for a fractional Schrödinger equation in which one seeks to recover information on an unknown potential by exterior measurements. In the talk, I prove uniqueness and stability of the "infinite data problem" and then address the recovery question. This also yields surprising insights on the uniqueness properties of the inverse problem in that it turns out that a single measurement suffices to uniquely recover the potential. - This is based on joint work with T. Ghosh, M. Salo and G. Uhlmann

Sebastian Schwarzacher (Universität Bonn)

Two Scale Solutions to Rate-Independent Systems

Abstract: Rate-independent systems are time-dependent PDEs, where the time derivative only occurs inside a positively zero-homogeneous dissipational force term. As such, these systems can be thought of as about 'half-way' between elliptic and parabolic systems. In particular, the evolution is quasi-static and jumps may occur in time if the elastic potential is non-convex. If one sees these processes as rescaling limits of parabolic equations, then it becomes clear that the jump evolution may be complicated and in fact needs to be carefully analyzed in order to prove well-posedness and the physically relevant energy dissipation balance. While there are several approaches to the analysis of rate-independent systems, so far questions of regularity, which are intimately tied to the development of a satisfactory solution theory, have not been considered in great detail. In this talk I will present some very recent results on existence, uniqueness, regularity, and approximation for rate-independent systems. Further, I will introduce what we call two speed solutions which allow to characterize the jump transients in an infinitesimal time variable. - The talk is about results that have been achieved in collaboration with F. Rindler, E. Suli, and J. Velazquez.

Ulisse Stefanelli (Universität Wien)

A Variational Resolution for Incompressible Navier-Stokes

Abstract: I will present a novel variational approach to the incompressible Navier-Stokes system by means of stabilized Weighted-Inertia-Dissipation-Energy (WIDE) functionals. These are parameter-dependent functionals on entire tarajectories whose minimization corresponds to an elliptic-in-time regularization of the system. By passing to the limit in the regularization parameter along subsequences of WIDE minimizers one recovers a classical Leray-Hopf weak solution. Time permitting, I will mention some other applications of this technique. - This is work in collaboration with Michael Ortiz and Bernd Schmidt.

Björn Stinner (University of Warwick)

Derivation and Analysis of a Cell Blebbing Model

Abstract: Cell blebs are protrusions of the cell membrane, which are instrumental for cell migration. Often, discrete particle and spring models are used to describe the mechanical aspects of the phenomenon. We present and discuss a continuum model that underpins such an approach where the deforming membrane is parametrised over a stationary surface representing the cell cortex. An abstraction of the model is analysed with regards to well-posedness. For this purpose, a finite element Galerkin approach is used that involves an approximation of the curved surface over which the problem is parametrised. A couple of numerical simulations will serve to illustrate the approach.

Contributed Talks:

Björn de Rijk (Universität Stuttgart)

Diffusive Stability of Planar Wave Trains in Reaction-diffusion Systems Against Nonlocalized Perturbations

Abstract: Planar wave trains are traveling wave solutions, whose wave profile p: $\mathbb{R}^2 \to \mathbb{R}^n$ are periodic in one spatial direction and constant in the transverse direction. They can be constructed by trivially extending a wave train on the real line. However, planar wave trains do not necessarily have the same stability properties as their underlying 1-dimensional counterparts: there is a sign criterion that determines whether spectral stability is `inherited'. Both in the 1- and 2-dimensional setting spectral stability yields diffusive stability against localized perturbations. In this talk, I show that in the planar case one can in fact allow for nonlocalized perturbations. I will explain the importance of 2 spatial dimensions for closing the nonlinear iteration argument. At the moment, an analogous result seems not to exist for the 1-dimensional setting, although nonlocalized perturbations affecting the phase of the wave train can be dealt with. - This is joint work with Björn Sandstede.

Tomas Dohnal (Martin Luther University Halle-Wittenberg)

First Order Coupled Mode Equations for Wavepackets in Higher Dimensions

Abstract: In nonlinear periodic media of arbitrary dimension d we consider the small amplitude asymptotics of wavepackets. The wavepackets have several N carrier Bloch waves of equal frequency. We use the cubic Gross-Pitaevskii equation as a prototype of the governing equation. The asymptotic scaling leads to first order coupled mode equations (CMEs) as amplitude equations. In the well understood one dimensional case the coupling of counter-propagating Bloch waves leads to CMEs which support a family of solitary waves parametrized by the velocity $v \in (-1,1)$ Can this be generalized to d dimensions such that in the CMEs a solitary wave family parametrized by $\vec{v} \in (-1,1)^d$ exists? Solitary waves are typically found in spectral gaps. For $d \geq 2$ at least 4 carrier waves are needed to produce CMEs with a spectral gap that supports solitary waves. However, only standing solitary waves have been found so far. We also provide a proof of the validity of the d-dimensional CME-asymptotics over asymptotically large time intervals.

Patrik Knopf (Universität Regensburg)

Weak Solutions of a Cahn-Hilliard System with Dynamic Boundary Conditions

Abstract: The Cahn-Hilliard equation is one of the most common models to describe phase separation processes of a mixture of two components. For a better description of short-range interactions of the material with the solid wall, various dynamic boundary conditions have been considered in recent times. New models with dynamic boundary conditions have been proposed recently by C. Liu and H.Wu[2]. We prove the existence of weak solutions to these new models by interpreting the prob lem as a suitable gradient ow of a total free energy which contains volume as well as surface contributions. The formulation involves an inner product which couples bulk and surface quantities in an appropriate way. We use an implicit time discretization and show that the obtained approximate solutions converge to a weak solution of the Cahn-Hilliard system in some suitable sense. This allows us to substantially improve earlier results which needed strong assumptions on the geometry of the domain.
[1]In collaboration with Harald Garcke, Dept. of Math., University of Regensburg, 93053 Regensburg.
[2]C. Liu, H.Wu, An Energetic Variational Approach for the Cahn-Hilliard Equation with Dynamic Boundary Conditions (2017), arXiv:1710.08318

Joel Kübler (Goethe-Universität Frankfurt)

Nonradial Bifurcation for the Henon Equation

Abstract: We study the nonlinear Henon equation on a ball and show that, as a parameter goes to infinity, nonradial solutions bifurcate from radial solutions. Our main tool is a thorough analysis of the eigenvalues of associated linearized operators which is based on a suitable rescaling of the equation. This allows us to identify a limit problem and yields asymptotic estimates for these eigenvalues.

Chris Kowall (Universität Heidelberg)

Shadow Limit Approximation for Reaction-Diffusion Systems

Abstract: Many models in mathematical biology consist of reaction-diffusion equations for which it is necessary to consider very different diffusion coefficients. Therefore it is worthy to consider a reduced model by letting the large diffusion coefficient tend to infinity. I will describe the notion of shadow limit in the case of a system of a semi linear heat equation coupled with an ordinary differential equation and give a rigorous proof of the convergence on finite time intervals. My recent work on long time intervals (depending on the diffusion coefficient) will also be considered. This is joint work with A. Marciniak-Czochra and A. Mikelic.

Maximilian Moser (Universität Regensburg)

Convergence of the Allen-Cahn Equation to Mean Curvature Flow with 90°-Contact Angle in 2D

Abstract: We consider the sharp interface limit of the Allen-Cahn equation with homogeneous Neumann boundary condition in a two-dimensional domain Ω, in the situation where an interface has developed and intersects $\partial\Omega$. Here a parameter $\varepsilon > 0$ in the equation, which is related to the thickness of the diffuse interface, is sent to zero. The limit problem is given by mean curvature flow with a 90°-contact angle condition and convergence using strong norms is shown for small times. Here we assume that a smooth solution to this limit problem exists on $[0, T]$ for some $T > 0$ and that it can be parametrized suitably. The strategy is as in Chen, Hilhorst, Logak [2] and Abels, Liu [1]: With asymptotic expansions we construct an approximate solution $(u_A^\varepsilon)_{\varepsilon\in(0,\varepsilon_o]}$ for the Allen-Cahn equation and estimate the difference of the exact and approximate solution with the aid of a spectral estimate for the linearized Allen-Cahn operator

$$-\Delta + \frac{1}{e^2} f''(u_A^\varepsilon(..t)) \text{ for } t \in [0, T\,].$$

Here the main new difficulty lies in the contact points. Therefore a suitable curvilinear coordinate system based on work of Vogel [3] is constructed. - Joint work with Helmut Abels

[1] H. Abels and Y. Liu. “Sharp Interface Limit for a Stokes/Allen-Cahn System”. In: Arch. Rational Mech. Anal. (2018), pp. 1–86. doi: 10.1007/s00205-018-1220-x.

[2] Chen, D. Hilhorst, and E. Logak. “Mass conserving Allen-Cahn equation and volume preserving mean curvature flow”. In: Interfaces and Free Boundaries 12 (2010), pp. 527–549.

[3] T. Vogel. “Sufficient conditions for capillary surfaces to be energy minima”. In: Pac. J. Math 194.2 (2000), pp. 469–489.

Wolfgang Reichel (Karlsruher Institut für Technologie)

Real-valued time-periodic localized standing waves for a class of nonlinear wave equations

We give a variational existence proof for time-periodic standing waves of two different 1+1 dimensional nonlinear wave equation with periodic potentials, namely
$-u_{xx} + V(x)u_{tt} = f(x,u)$, and $-u_{xx} + V(x)u_{tt} = \Gamma(x)(u_t^3)_t$.

These waves are localized in the unbounded spatial direction. Using Fourier decomposition in time we can solve the resulting variational problem via constrained or direct minimization. In the first of the two cases it appears that the admissible growth of the nonlinearity f is limited by a Sobolev-exponent introduced via the regularity of the potential V. In the second of the two cases we also present some numerical approximations. This is joint work with Andreas Hirsch and Simon Kohler (Karlsruhe Institute of Technology, Germany).

Roland Schnaubelt (Karlsruher Institut für Technologie)

Decay of Quasilinear Maxwell Equations with Conductivity

Abstract: We discuss the Maxwell system with nonlinear instantaneous material laws and a strictly positive conductivity in the domain or at the boundary. The coefficients are matrix-valued. For small initial data we can show that the solution exponentially decays to 0 in H^3. Besides recent local well-posedness results, we use higher order energy bounds and observability-type estimates both with error terms arising from the quasilinearity. This is joint work with Irena Lasiecka (Memphis), Michael Pokojovy (El Paso) and Martin Spitz (Karlsruhe).

Sebastian Throm (Technische Universität München)

Long-Time Behaviour for Smoluchowski's Coagulation Equation with Cluster In- and Output

Abstract: The classical Smoluchowski coagulation equation is a mean field model which describes the evolution of a system of aggregating particles. It is in frequent use in applications in very different areas and also on a very broad spectrum of length scales, ranging from microscopic phenomena up to the formation of stars. In this talk, we will consider a modification of this model where also in- and output of clusters takes place. For this equation, we will address the well-posedness of solutions and we will give conditions on the coefficients under which the system, in the long-time limit, converges towards an equilibrium.

Timo Weidl (Universität Stuttgart)

Edge Resonances in Elastic Media with Zero Poisson Coefficient

Abstract: A two-dimensional elastic semistrip with stress-free boundary conditions and zero Poisson coefficients has an embedded eigenvalue on top of the continuous spectrum. This effect is known as edge resonance. For an infinite plate of finite thickness with a drilling hole $(\mathbb{R}^2 \setminus \Omega) \times \mathrm{I}$ actually infinitely many edge resonances will occur. This is related to the spectral problem of perturbations of symbols with strongly degenerated minima, which also appear in BCS theory. Recently new methods for dealing with perturbations by boundary conditions (cracks) have been developed. I give an overview on some of our results in this area, which still poses a number of mathematical challenges.