Second GAMM-Workshop on

Analysis of Partial Differential Equations

Universität Stuttgart, September 29 – October 1, 2014

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Speaker: **Anton Arnold**
Title of Talk: **Entropy method for hypocoercive Fokker-Planck equations with linear drift**
Scheduled: **Monday, 29 September 2014, 14:05–14:50**

Abstract: In the last 15 years the *entropy method* has become a powerful tool for analyzing the large-time behavior of the Cauchy problem for linear and non-linear Fokker-Planck type equations (advection-diffusion equations). These problems appear as homogeneous versions of kinetic Fokker–Planck models, porous media equations, or electron transport models for semiconductor devices, e.g. In particular, this entropy method can be used to analyze the rate of convergence to the equilibrium, and in parallel to prove the validity of logarithmic Sobolev inequalities and a variety of Poincaré-type inequalities. The essence of the method is to first derive a differential inequality between the first and second time derivative of the relative entropy, and then between the entropy dissipation and the entropy.

For degenerate parabolic equations, the entropy dissipation may vanish for states other than the equilibrium. Hence, the standard entropy method does not carry over. For hypocoercive Fokker-Planck equations, we first establish a condition that is equivalent to the existence of a unique normalised steady state. By introducing an auxiliary functional (of entropy dissipation type) we prove the exponential decay of the solution towards the steady state in relative entropy. Finally, we show that the obtained rate is indeed sharp (both for the logarithmic and quadratic entropy).

References:

[1] J. Erb, A. Arnold. Sharp entropy decay for hypocoercive Fokker-Planck equations with linear drift, preprint 2014.

[2] A. Arnold, P. Markowich, G. Toscani, A. Unterreiter. On logarithmic Sobolev inequalities and the rate of convergence to equilibrium for Fokker-Planck type equations. Comm. PDE 26/1-2 (2001) 43-100.

Speaker: **Lisa Beck**
Title of Talk: **Regularity and uniqueness for BV-minimizers**
Scheduled: **Wednesday, 1 October 2014, 9:00–9:45**

Abstract: For a smooth function $u\colon \Omega \to \mathbb{R}$ the n-dimensional area of its graph over a bounded domain $\Omega \subset \mathbb{R}^n$ is given by

$$\int_\Omega \sqrt{1+|Du|^2}\,dx\,.$$

A natural question is whether or not minimizers of this functional exist among all functions taking prescribed boundary values. It turns out that existence is guaranteed only in a generalized sense. For this purpose the functional is extended suitably to the space of functions of bounded variation via relaxation, where for instance attainment of the prescribed boundary values is not mandatory, but non-attainment is penalized. As a further consequence of the relaxation, generalized minimizers are not necessarily unique.

In my talk I will discuss similar convex variational integrals under a linear growth condition and investigate this phenomenon of non-uniqueness in more detail. In particular, I will explain why the regularity of generalized minimizers plays a crucial role for this issue, and how one benefits from an alternative approach to the minimization problem by means of its dual problem in the sense of convex analysis. The results presented in this talk were obtained in collaboration with Thomas Schmidt (Erlangen).

Speaker: **Karoline Disser**
Title of Talk: **Asymptotic behaviour of a rigid body with a cavity filled by a viscous liquid**
Scheduled: **Wednesday, 1 October 2014, 11:15–11:45**

Abstract: We discuss the system of equations modeling the free motion of a rigid body with a cavity filled by a viscous (Navier-Stokes) liquid. The aim is to provide a rigorous proof of Zhukovskiy's Theorem which states that in the limit $t \to \infty$, the relative fluid velocity tends to zero and the rigid velocity of the full structure tends to a steady rotation around one of the principle axes of inertia. In particular, we show that every (reasonable) weak solution is subject to Zhukovskiy's Theorem – so that it applies to finite-energy initial data of arbitrary size. Using previously known results, the existence of weak solutions of this type can be obtained.

Independently of the geometry and of parameters, this shows that the presence of fluid prevents precession of the body in the limit. In general, we cannot predict which axis will be attained, but we show stability of the largest axis and provide criteria on the initial data which are decisive in special cases.

For this system, it is straightforward to show that the kinetic energy provides a strict Lyapunov functional for (regular) solutions, whereas total angular momentum is preserved. The main point in the proof is to show that despite this conservative aspect and in the absence of stability, weak solutions become regular eventually. For large-time trajectories, a suitable version of LaSalle's invariance principle then applies.

Speaker: **Julian Fischer**
Title of Talk: **Optimal estimates on free boundary propagation for the thin-film equation**
Scheduled: **Monday, 29 September 2014, 14:55–15:40**

Abstract: We present a method for the derivation of lower bounds on free boundary propagation for the thin-film equation, one of the most prominent examples of a higher-order degenerate parabolic equation. In particular, we obtain sufficient conditions for instantaneous forward motion of the free boundary, upper bounds on waiting times, as well as lower bounds on asymptotic propagation rates. Our estimates coincide (up to a constant factor) with the previously known reverse bounds and are therefore optimal. To the best of our knowledge, these results constitute the first lower bounds on free boundary propagation for any higher-order degenerate parabolic equation. Our technique is based on new monotonicity formulas for solutions to the thin-film equation which involve weighted entropies with singular weight functions. It turns out that our method is not restricted to the thin-film equation, but also applicable to other higher-order parabolic equations like quantum drift-diffusion equations.

Speaker: **Carolin Kreisbeck**
Title of Talk: **Thin-film limits of functionals on $\mathcal{A}$-free vector fields**
Scheduled: **Tuesday, 30 September 2014, 11:15–11:45**

Abstract: Working with variational principles subject to linear PDE constraints conveyed by a constant-rank operator $\mathcal{A}$ allows us to treat a number of problems in continuum mechanics and electromagnetism in a unified way. The topic of this talk, which reports on a joint work with Filip Rindler (University of Warwick), is 3d-2d dimension reduction within this general $\mathcal{A}$-free framework. We study the effective behavior of integral functionals as the thickness ε of the domain tends to zero. Under certain conditions we show that the Γ-limit is an integral functional and give an explicit formula. The limit functional turns out to be constrained to $\mathcal{A}_0$-free vector fields, where the limit operator $\mathcal{A}_0$ is in general not of constant rank. While the lower bound follows from a Young measure approach together with a new decomposition lemma, the construction of a recovery sequence relies on algebraic considerations in Fourier space. This part of the argument requires a careful analysis of the limiting behavior of the rescaled operators $\mathcal{A}_\varepsilon$ by a suitable convergence of their symbols, as well as an explicit construction for plane waves. As applications, we characterize a thin-film Γ-limit in micromagnetics and recover the energy of a membrane model with bending moment in nonlinear elasticity.

Speaker: **Bogdan-Vasile Matioc**
Title of Talk: **Qualitative properties of rotational waves**
Scheduled: **Tuesday, 30 September 2014, 14:20–15:05**

Abstract: We discuss the symmetry and regularity properties of two-dimensional periodic water waves traveling under the influence of gravity. The situation considered is that of rotational waves possessing a general vorticity function, but we exclude the presence of stagnation points, that is that fluid particles that travel horizontally with the same speed as the wave. We first present a remarkable result for waves with rough, that is discontinuous or even unbounded, vorticity: the wave surface and all other streamlines are real-analytic graphs. This nice regularity property together with sharp elliptic maximum principles can be used to prove that the symmetry of the wave is an intrinsic property of the flow. More precisely, we show that a wave is symmetric and has a single crest and trough per period if and only if there exists a vertical line within the fluid domain such that all the fluid particles located on that line minimize there simultaneously their distance to the fluid bed as they move about.

This is joint work with Joachim Escher and Anca Matioc.

Speaker: **Josef Málek**
Title of Talk: **On elastic solids with limiting small strain: Modeling, analysis and computation**
Scheduled: **Wednesday, 1 October 2014, 14:15–15:00**

Abstract: In order to understand nonlinear responses of materials to external stimuli of different sort, be they of mechanical, thermal, electrical, magnetic, or of optical nature, it is useful to have at one's disposal a broad spectrum of models that have the capacity to describe in mathematical terms a wide range of material behavior. It is advantageous if such a framework stems from a simple and elegant general idea. Implicit constitutive theory of materials provides such a framework: while being built upon simple ideas, it is able to capture experimental observations with the minimum number of physical quantities involved. It also provides theoretical justification in the full three-dimensional setting for various models that were previously proposed in an ad hoc manner. From the perspective of the theory of nonlinear partial differential equations, implicit constitutive theory leads to new classes of challenging mathematical problems. This lecture focuses on implicit constitutive models for elastic solids in general, and on its subclass consisting of elastic solids with limiting small strain. After introducing the basic concepts of implicit constitutive theory, we provide an overview of results concerning modeling within the framework of continuum mechanics. We then concentrate on the mathematical analysis of relevant boundary-value problems associated with models with limiting small strain, and we present the analytical results concerning the existence of weak solutions in general three-dimensional domains. Finally, we show numerical simulations concerning anti-plane stress problems in V-notched (i.e., non-convex) domains.

The lecture is based on a joint results with Miroslav Bulíček, Vojtěch Kulvait, K. R. Rajagopal, Endré Süli and Jay Walton.

Speaker: **Alexander Mielke**
Title of Talk: **Homogenization of parabolic gradient systems via evolutionary Γ-convergence**
Scheduled: **Tuesday, 30 September 2014, 9:00–9:45**

Abstract: We consider a family of gradient systems $(\mathbf{H}, \mathcal{E}_\varepsilon, \mathcal{R}_\varepsilon)$ on a Hilbert space $\mathbf{H}$, where the dissipation potential $\mathcal{R}_\varepsilon$ is quadratic and does not depend on the state, i.e. $\mathcal{R}_\varepsilon(u, \dot u) = \frac{1}{2}\langle \mathbb{G}_\varepsilon \dot u, \dot u\rangle$:

$$\mathbb{G}_\varepsilon \dot u = -\mathrm{D}\mathcal{E}_\varepsilon(u) \ \text{ in } \mathbf{H}^* \qquad \Longleftrightarrow \qquad \dot u = -\nabla_{\mathbb{G}_\varepsilon}\mathcal{E}_\varepsilon(u). \tag{G_ε}$$

The general question of evolutionary Γ-convergence (cf. [Mie14]) is concerned with the question in what sense the pair $(\mathcal{E}_\varepsilon, \mathcal{R}_\varepsilon)$ should converge to limit functionals $(\mathcal{E}_0, \mathcal{R}_0)$ such that the solutions $u_\varepsilon : [0,T] \to \mathbf{H}$ of (G_ε) converge to the solutions u_0 of (G_0).

Using the concept of uniform λ-convexity, i.e. $\mathcal{E}_\varepsilon - \lambda\mathcal{R}_\varepsilon$ is convex, one can rewrite G_ε as an evolutionary variational estimate

$$\begin{aligned}&\forall\, 0 \le s < t\ \forall\, w \in \mathbf{H} :\\ &\mathrm{e}^{\lambda(t-s)}\mathcal{R}_\varepsilon(u(t){-}w) - \mathcal{R}_\varepsilon(u(s){-}w) \le \frac{\mathrm{e}^{\lambda(t-s)}-1}{\lambda}\Big(\mathcal{E}_\varepsilon(w) - \mathcal{E}_\varepsilon(u(t))\Big).\end{aligned} \tag{EVE}$$

We show that we can pass to the limit in this equation if we have the Mosco convergence $\mathcal{E}_\varepsilon \xrightarrow{\mathrm{M}} \mathcal{E}_0$ in $\mathbf{H}$ and the continuous convergence $\mathcal{R}_\varepsilon \xrightarrow{\mathrm{c}} \mathcal{R}_0$ in $\mathbf{H}$.

We give applications to the homogenization of parabolic equation $g_\varepsilon \dot u = \mathrm{div}\,\big(A_\varepsilon \nabla u\big) + b_\varepsilon u$ and to the Cahn-Hilliard equation

$$\dot u = \mathrm{div}\,\big(M_\varepsilon \nabla \mu\big) \quad \text{with } \mu = \mathrm{D}\mathcal{E}_\varepsilon(u) = \partial_u F_\varepsilon(u) - \mathrm{div}\,\big(A_\varepsilon \nabla u\big).$$

We show that it is enough that the initial conditions satisfy $u_\varepsilon(0) \to u_0(0)$ in $\mathrm{H}^{-1}(\Omega)$ to conclude the convergence $u_\varepsilon(t) \rightharpoonup u_0(t)$ in $\mathrm{H}^1(\Omega)$ for all $t > 0$.

Finally, we discuss the homogenization of the temperature-dependent phase-field model of Penrose-Fife. The theory is nicely applicable if we use the internal energy instead of the temperature as the unknown.

This talk is based on work with Sina Reichelt and Matthias Liero and with Ulisse Stefanelli.

References:

[Mie14] A. Mielke. On evolutionary Γ-convergence for gradient systems. *WIAS Preprint 1915*, 2014.

Speaker: **Mark A. Peletier**
Title of Talk: **Large deviations, gradient flows, and taking limits**
Scheduled: **Tuesday, 30 September 2014, 13:30–14:15**

Abstract: It is now well understood that there is a strong connection between gradient flows on one hand and large-deviation principles on the other hand. In a sense, this connection takes the form of a single functional that characterizes both the large deviations and the gradient-flow behaviour.

In this talk, which is work with Giovanni Bonaschi and Giuseppe Savare, I will show how this insight produces a unification of both structure and method. I will focus on a very simple stochastic system, and show how the large-deviation rate functional is related to a generalized gradient flow - with a parameter. By taking various limits in this parameter we recover both linear gradient-flow behaviour and rate-independent behaviour.

The unification lies in the fact that we can base our entire discussion on this one functional. It characterizes the structure and is also the main actor in the limit-taking, leading both to compactness and to characterization of the limit. This work shows how the connection between large deviations and gradient flows is not only philosophically interesting but also provides tools for analysis.

Speaker: **Jens Rademacher**
Title of Talk: **Pattern formation in Landau-Lifshitz-Gilbert-Slonczewski equations with aligned fields**
Scheduled: **Wednesday, 1 October 2014, 11:45–12:15**

Abstract: The self-organized emergence of spatio-temporal patterns is a ubiquitous phenomenon in nonlinear processes on large homogeneous domains. In this talk a class of Landau-Lifshitz-Gilbert-Slonczewski equations is studied from this viewpoint, highlighting various aspects of the theory. The model describes magnetization dynamics in the presence of an applied field and a spin polarized current. Here we consider the case of axial symmetry and focus on coherent structure solutions that occur due to the symmetry.

This is joint work with Christof Melcher (RWTH).

Speaker: **Nikolaos Roidos**
Title of Talk: **The porous medium equation on manifolds with conical singularities**
Scheduled: **Monday, 29 September 2014, 17:15–17:45**

Abstract: By using earlier results on the existence of bounded imaginary powers for cone differential operators on higher order Mellin-Sobolev spaces $\mathcal{H}_p^{s,\gamma}(\mathbb{B})$, over a manifold with conical singularities, we show existence of a short time solution for the porous medium equation with the help of a theorem by Clément and Li. We also obtain short time asymptotics of the solution near the conical point.

Speaker: **Jürgen Saal**
Title of Talk: **Some progress on fluid dynamics in singular domains**
Scheduled: **Wednesday, 1 October 2014, 9:50–10:35**

Abstract: Fluid flow in singular domains has quite some significance in applications. For instance, models of cyclones, such as hurricanes or tornados, are considered in a cylinder which is, by the presence of edges, a so-called weakly singular domain. Another important application is given by wetting and de-wetting phenomena. Here in general three-phase dynamic contact lines (gas/fluid/solid, fluid/fluid/solid, etc.) appear, which at the end lead to systems of equations on wedge type domains. In spite of their significance, there still exist fundamental open problems concerning their rigorous mathematical treatment. The aim of the lecture, hence, is to present some recent progess in this direction. We intend to present maximal regularity type results for the Stokes equations subject to Navier slip on cylindrical domains, on wedge type domains, and on (graph) Lipschitz domains. These results are based on parabolic cylindrical theory, the commuting and non-commuting operator sum method, and off-diagonal estimates. As a consequence of the linear outcome we can recover well-known results for the Navier-Stokes equations obtained on smooth domains.

Speaker: **Veronika Schleper**
Title of Talk: **The compressible to incompressible limit of the 1D isothermal Euler equations in a non-smooth setting**
Scheduled: **Monday, 29 September 2014, 16.15–16:45**

Abstract: We discuss the limit of compressible to incompressible Euler equations in the context of two-phase flows. Hereby, the limit is obtained in the liquid phase, while the compressibility of the gas phase is preserved. The starting point is the fully compressible setting, given by the isothermal Euler equations

$$\begin{aligned}\partial_t\rho + \partial_x(\rho v) &= 0\\ \partial_t(\rho v) + \partial_x(\rho v^2 + p_i(\rho)) &= 0\end{aligned}$$

with pressure laws $p_i(\rho)$ chosen separately in each phase. To close the model and determine the behaviour at the fluid-fluid interface, we impose a no-mass transfer condition. Mathematically, this yields the jump condition

$$[[\rho(s-v)]] = 0,$$

where s is the interface velocity and $[[a]] := a_+ - a_-$ denotes the jump across the interface.

We investigate the behaviour of the Riemann problem at the interface as the speed of sound in one of the phases tends to $+\infty$. During the analysis, it turns out that the incompressible limit of the Riemann problem is not helpful neither for the analysis of droplet behaviours nor for simulation purposes. We discuss the physical reason for this behaviour and show that we can overcome this seemingly unphysical behaviour when the full Cauchy problem is considered.

In the second part of the talk, we show how weak solutions of the fully compressible two-fluid Cauchy problem depend on the sound speed in the liquid phase. From these observations, we construct a set of initial data, such that the solution to the fully compressible model exists for all (sufficiently large) liquid sound speeds. This result at hand, we can go to the incompressible limit in the liquid phase and obtain the convergence of the semigroup of fully compressible solutions to the semigroup constructed by a coupling of compressible and incompressible Euler equations. This limit is given in L^1_{loc} for the density and velocity, while the convergence of the pressure is obtained only in a weak*-sense in L^∞.

References:

[1] R. M. Colombo, V. Schleper: Two-phase flows: non-smooth well posedness and the compressible to incompressible limit, *Nonlinear Anal. Real World Appl.* 13(2012), pp. 2195-2213.

[2] R. M. Colombo, G. Guerra, V. Schleper: The compressible to incompressible limit of 1D Euler equations: the non-smooth case, *Preprint* 2013, available on arXiv (`http://arxiv.org/abs/1308.4109`).

Speaker: **Ben Schweizer**
Title of Talk: **Waves in heterogeneous media: derivation of dispersive limit models**
Scheduled: **Tuesday, 30 September 2014, 9.50–10.35**

Abstract: We investigate wave propagation in a heterogeneous medium. A disturbance (the deformation in an elastic wave or the magnetic field in an electro-magnetic wave) is described by the second order wave equation in divergence form without diffusion, $\partial_t^2 u^\varepsilon = \nabla \cdot (a_\varepsilon \nabla u^\varepsilon)$. The coefficient is of the form $a_\varepsilon(x) = a(x/\varepsilon)$ with a periodic function $a : \mathbb{R}^n \to (0, \infty)$, such that the medium has heterogeneities on the scale ε. According to results from the 1990s, on time scales of order 1, the effective properties of the medium are well-described by the homogenized wave equation $\partial_t^2 u = \nabla \cdot (a_* \nabla u)$. On the other hand, when large time intervals of order ε^{-2} are considered, numerical results show dispersion of a wave packet. We demonstrate an analysis of the problem with Bloch-waves and derive a dispersive effective equation that describes u^ε on large time scales. Numerical tests show the effectiveness of the limit system in one- and two-dimensional examples.

This is joint work with T. Dohnal and A. Lamacz.

Speaker: **Christian Seis**
Title of Talk: **On long-time asymptotic expansions for slow diffusion equations**
Scheduled: **Wednesday, 1 October 2014, 13:45–14:15**

Abstract: In this talk, I describe recent advances in the understanding of the long-time behavior of certain nonlinear diffusion equations, including the porous medium equation, the parabolic p-Laplace equation, and the thin-film equation. These equations have in common that the long-time dynamics of any solution is governed by a self-similar solution. Optimal rates of the convergence to self-similarity are known. In a series of works (partially with J. Denzler and R. McCann), we go one step further: We linearize the equation about the self-similar solution and compute the complete spectrum and the corresponding eigenfunctions of the differential operator. This knowledge can be used to (at least heuristically) predict the long-time asymptotic expansions of solutions.

Speaker: **Jens Wirth**
Title of Talk: **Dispersive estimates for Fourier integrals and applications to hyperbolic systems**
Scheduled: **Tuesday, 30 September 2014, 11:45–12:15**

Abstract: The talk gives an overview on dispersive type estimates for certain classes of Fourier integrals and discusses possible applications to hyperbolic systems. The results presented are based on the author's papers [2] and [3], and the material presented in [1].

This is joint work with Michael Ruzhansky.

References

[1] M. Ruzhansky, J. Wirth, *Asymptotic Behaviour of Solutions to Hyperbolic Equations and Systems*, in D. Cruz-Uribe, A. Fiorenza, M. Ruzhansky, J. Wirth, S. Tikhonov (ed.) *Variable Lebesgue Spaces and Hyperbolic Systems*, Advanced Courses in Mathematics CRM Barcelona, Vol 27, ISBN 978-3-0348-0839-2, Birkhäuser, 2014

[2] M. Ruzhansky, J. Wirth, *Dispersive estimates for hyperbolic systems with time-dependent coefficients*, Journal of Differential Equations 251 (2011) 941–969

[3] M. Ruzhansky, J. Wirth, *Dispersive type estimates for Fourier integrals and applications to hyperbolic systems*, Discrete and Continuous Dynamical Systems, 2011 Supplement, 1263–1270.

Speaker: **Dominik Zimmermann**
Title of Talk: **Approximation theorems for pattern forming systems with a conserved quantity**
Scheduled: **Monday, 29 September 2014, 16.45–17.15**

Abstract: The Ginzburg-Landau equation can be derived formally by a multiple scaling ansatz in order to describe small solutions of a pattern forming system, when a homogeneous steady state becomes unstable via a Turing instability. That there are indeed solutions that behave as predicted by the Ginzburg-Landau equation is then made rigorous by an approximation theorem. If the pattern forming system possesses a conserved quantity, similar formal computations lead to a Ginzburg-Landau-like system of evolution equations. We show that an analogous approximation result holds by extending the existing proofs for the Ginzburg-Landau approximation to cover the case when a conserved quantity is present. Since our considerations are motivated by Bénard-Marangoni convection, we also demonstrate how to handle the case of quasilinear problems.